EYFS- Progression in Calculation

Comparison

Skill	Small focus	Potential difficulties	Possible teaching strategies
<u>Vocabulary</u> • Comparing the size of quantities (by looking at the quantity of objects, not their size or orientation)	• Recognise when two **quantities** different by saying; - Which of two **quantities** is more - Which of two **quantities** is fewer • Recognise when two **quantities** are the same	• Refers to any number of objects more than one as 'lots' • Assumes that the amount of space taken up by a group of objects relates to the overall quantity, e.g. thinks that a row of five large objects is more than a row of eight small objects • Fails to recognise that a group of four apples contains the same number as a group of four peas • Is insecure with the language of comparison, e.g. can say 'Pradeep has more shells than Jo' when Pradeep had five and Jo has three, but not 'Jo has fewer than Pradeep'	• Ensure that children have frequent opportunities to compare sets of objects of different sizes. • Provide opportunities for children to recognise small groups as equal / same even when arranged differently or spread out • Give children strategies for comparing the number of objects in different groups, e.g. pairing dissimilar objects from each set of counting objects in each set • Model the language of comparison in everyday situations and ensure children are asked to identify who has fewer as well as who has more and encourage children to use the corresponding language
<u>Estimation</u> • Make a reasonable estimation (sensible guess) of a number of objects without counting	• Recognise if the quantity in a set is known (subitised) or unknown • Recognise known images within a quantity and use to support estimation (structured arrangement)	• Believes that an estimation has to match the exact number of objects • Makes wild estimates, e.g using favourite large numbers	• Make every day examples of estimation used in all areas of learning explicit- 'do you think we've got enough cups?' • Do not as children to estimate objects that they can subitise or count very quickly

	• Recognise known images within a quantity and use to support estimation (random arrangement)		• Ask children to estimate mixed steps of objects, sounds (e.g shake a tin can containing coins) and objects they cant easily see (e.g the number of objects in a pencil case) • Model estimation strategies for children e.g. 'do you think there ae more or fewer than 10?; ensure they have opportunity to build experience, for example, collecting sets of ten objects and using these as a benchmark when making other estimates.
<u>Prediction</u> • Predict the number that comes after or before in a number rhyme or problem	• Recognise if a number rhyme, story or problem is increasing • Recognise if a number rhyme, tory or problem is decreasing	• Doesn't link the rhyme with being a number problem • Can't identify if the number rhyme/ problem has increased or decreased in quantity • Predicts one more rather than one less (fewer) • Predicts one less (fewer) rather than one more	• Sing familiar number rhymes with the children using props and/ or fingers and/ or numerals to help aid their understanding • Use real life examples of number problems solving as they arise in children's play where they can predict the answer
Ordering • Order numbers (quantities, numerals and mixed representations)	• Identify the smallest quantity to start with • Identify the largest quantity to start with • Identify zero to start with	• Has to count from 1 to find the number before or after a given number as they are insecure when counting from other starting numbers • Is unable to identify a missing number within a given range	• Give children experience of counting from different starting points, both forwards and backwards • Provide children with partially completed number tracks, e.g 1, , , ,5, to give them experience of

		• Confuses the number that comes before and after a given number, e.g. saying four comes before three rather than after three • Confuses the language of ordering numbers, e.g says 'six comes behind seven'	identifying which number goes before or after given numbers. • Provide children with a selection of numbers or numerals to order on an empty track e.g 2, 5, 9, 10 • Provide images of objects, e.g. images of the spots on a dice, and ask children to order these and identify the image that comes before or after • Order combination of numbers and numerals
Counting			
Skill	**Small focus**	**Potential difficulties**	**Possible teaching strategies**
<u>Verbal counting</u> • Know the number names in order • Chanting forwards and backwards in ones • Recognising a chanting sequence	• Practise chanting forwards from zero • Practise counting from number to zero • Recognise if a chanting sequence is increasing or decreasing • Practise chanting forwards from any starting point • Practise chanting backwards from any starting point	• Doesn't separate the number names but says 'onetwothreefour…' as if one word • Misses out number names, e.g. one, two, three, five, six • Repeats number names, e.g- one, two, two, three, four • Uses the correct number names but in the wrong order, e.g one, two, three, four, six, five • Recites accurately the number names when starting from one or zero but has difficulty	• Encourage children to rehearse number names and order using songs, games, books and rhymes • Give children experience of chanting in different starting points, both forwards and backwards changing the pace of chanting • Encourage children to spot the mistake a puppet makes when chanting and to teach the puppet how to chant correctly • Ensure that the children have frequent opportunities to cross

		counting from other starting numbers and when counting backwards • Confuses two sequences e.g- ten, eleven, twelve, thirty, forty, fifty, sixty, seventy, eighty, ninety • Confuses the vocabulary of counting 'backwards', 'forwards', 'up', 'down', 'on', 'back from', etc. and counts in the wrong direction	tens boundaries in chanting activities, rhymes and games • Use a large number track on the floor, which children can jump along while chanting forwards and backwards • Consider horizontal and vertical prompts to support chanting.
<u>Subitizing</u> • Recognise small numbers (<3 or 4) of objects without counting	• Recognising a small quantity where all the object are the same • Recognising a small quantity where a variable differs between objects • Recognise a small quantity where all objects are different	• Can only subitise when objects are arranged in familiar patterns • Finds it difficult to subitize when objects are different colours, size or shape • Relies on counting small number of objects even when they could recognise how many there are	• Encourage instant recognition of one, two or three dots by providing dice, playing cards and dominoes for the children to include in their play • Put small numbers of objects in unfamiliar patterns and compare with familiar patterns, such as spots on dice and dominoes • When children subitise correctly, accept their responses rather than insisting that they check by counting • Encourage children to visualise common familiar patterns, e.g looking closely at the arrangements of dots on a dice, talking about what they can see,

			then hiding the dice and asking children to recreate the pattern
<u>Recognising</u> • Identifying number representations, including numerals	• Introduce familiar, structured representations (dice and ten frames etc.) • Using marks, begin to record own representations of numbers • Identify familiar patterns of up to six objects • Introduce numerals in different sizes and fonts shown alongside number images • Represent numbers using fingers • Identify numerals in isolation • Match numeral to a correct number representation • Begin to form numerals • Knows that objects can be counted in any order, from any starting point, as long as every item is counted once (Order irrelevance principle) • Identifies that the number associated with the last object touched is the total number of	• Does not coordinate number names with items being counted • Coordinates number names with items counted but misses out one or more of the objects • Coordinates number names with items counted but counts an object more than once • Does not associate the number names said with the number of objects counted • Does not realise that the last number in a count is the number in the set • When asked for the total in a collection that they have just counted, counts the whole set again • Loses track when counting objects that are randomly arranged, or that cannot be touched • Loses track when counting sounds	• Model touch counting each object as you say a number • Use 2D and 3D shape features as counting opportunities • Model counting objects systematically, e.g. point to one at a time from top to bottom, left to right • Remember to vary and mix resources that children are counting • When counting objects, discuss with the children what they could do to make counting easier, e.g- put them in a line or familiar representation, move them as we count, lace into a structured scaffold (ten frame/ Hungarian ten frame) • Use tidy up time to reinforce counting objects • Count things that can be seen but not touched, e.g. model pointing at pictures n the wall to count how many there are • Model how you can cover static objects with counters to help keep track of what has been

	objects (Cardinal principle) • Know that zero is the cardinal value of an empty set • Count out a smaller number of objects from a larger group • Count objects that cannot be moved (Abstract principle) • Count actions or sounds (Abstract principle) • Knows that zero is a place holder		counted • Count sounds, e.g. drop coins into an empty tin at irregular intervals to encourage children to wait for each sound before it is counted. • Appreciate that numbers can identify how many objects are in a set • Know that the last number in the count gives the total
<u>Ordinal and Nominal</u> • Using ordinal numbers			• Use first, second and third in practical contexts • Use in a range of contexts • Make number books that have meaning for the children, such as favourite numbers, birthdays or telephone numbers
Composition			
Skill	**Small Focus**	**Potential Difficulties**	**Possible Teaching Strategies**
<u>Part-Whole relationships</u> • Understanding the connection between an object and how it might be split	• Understand that a whole can be one object and if some of it is remover, it is no longer the original whole • Understand that if a whole is split the new parts are smaller than the original whole	• Children might think that parts must be equal	

	• Understand the smaller parts can be put back together to make the original whole • Understand that a group of objects can represent a whole (unitising)		
<u>Numbers within Numbers</u> • Knowing that each number builds upon the previous number • Knowing that a number can be partitioned into smaller numbers • Knowing numbers can be combined to form a larger number	• Show a number as the previous number plus one (Number System) • Understand that a number can be partitioned into smaller quantities (Decomposition) • Understand that quantities can be combined to make a number (Composition)		• Give children plenty of experience of finding for themselves that the total number of objects stays the same (Conservation of Number) • Use bead-strings to support • Use fingers to represent a number in different ways • Use bunny fingers (fingers on top of your head) when asking children to show quantities on their fingers • Begin to use marks to record their understanding • Use this opportunity to raise awareness of doubles and halves • A PPW model may be useful
<u>Conceptual Subitizing</u> • Recognise a quantity as a whole from know or subitized parts	• Know a structured representation of a whole by describing two parts		• Provide children opportunities to make numbers on structured scaffolds such as Ten Frames or Hungarian Frames. Discuss the composition of the numbers

	• Know a structured representation by describing what is absent • Recognise or subitize quantities in unstructured arrangements • Recall know number facts to identify whole in an unstructured arrangement		• Encourage children to find known representations of numbers in familiar manipulatives, such as laying cards and dominoes • To support children, use a different coloured to distinguish a known representation in an unstructured arrangement
<u>Unitizing and Making Ten</u> • Rename a group of objects as one unit	• Collect together a set of objects and rename as one unit • Know that multiple units must have the same amount in each • Count number of units • Know teen numbers as one unit of ten and…	• When making a unit the children still see the number as how many are in the group rather than see the group as 1 unit • Children counting all objects not counting the units	• Use familiar units e.g pairs of socks, shoes • Provide lots of opportunities for the children to unitize smaller numbers within their play, not us making groups of 10, e.g. 3 sweets in each bag, 6 pieces of pasta in each bowl, 8 flowers in each bunch etc.
Change			
Skill	**Small Focus**	**Potential Difficulties**	**Possible Teaching Strategies**
<u>Combining Sets</u> • Understand that adding can be the change of sets into one new group (Aggregation)	• Recognise that sets can be combines to make a new group • Recognise when a contextual problem is following this addition structure • When needed, generate a collective noun for the new combined group • Represent the sets from a contextual problem using manipulatives	• Some children may not be able to identify a collective noun to combine the sets under	• Always use context for partitioning • Use proper to enhance number stories • Remember to use similar categories, e.g. I have five fruit. Three are apples, how many are pears? • If the total is below 4, allow children to subitize. There is no need for them to count.

	• Use the vocabulary of addition when combining the sets into a new group • Find the total quantity of a new group by using a strategy • Know the new group is a larger quantity than either of the original sets • Begin to use marks to record their understanding of aggregation		• Encourage children to count all objects in the new group to find the total, if it is 5 or above • When children are ready, promote the idea of counting on from one of the sets. Support this idea with the use of familiar structured representations. Objects within a set could be covered or replaced with numerals. • Encourage children to count on from the set with the largest quantity as an effective strategy • Vary types of question to include missing number problems. Scaffold this by showing the scenario with props and then covering a set over • Make explicit the link between one more and adding one
<u>Separating Sets</u> • Understand that subtraction can be the change of a group by separating a set to identify what is remaining (Partition)	• Recognise that a group can be separated into sets • Recognise when a contextual problem is following this subtraction structure • Give the separated sets and remaining set each a noun • Represent the group from a contextual problem using		• Always use a context for partitioning • Use props to enhance number stories • Remember to use similar categories, e.g I have five fruit. Three are apples. How many pears?

	manipulatives • Use the vocabulary of subtractions when separating a set from the group • Find the quantity of the remaining sets by using a strategy • Know that both of the sets are smaller quantities than the original group • Begin to use marks to record their understanding of partitioning		• If hat is remaining is below 4, allow children to subitize. There is no need for them to count • Encourage children to count what is remaining, if it is 5 or above • When children are ready, promote the idea of counting back. Scaffold this using structured frames • Vary types of question to include missing number problems. Scaffold this by showing the scenario with props and then covering a set over • Make explicit the link between one fewer and subtracting one
<u>Increasing a Set</u> • Understand that adding can be the change of an existing set by increasing its quantity (Augmentation)	• Recognising that if objects are added to am existing set, the quantity has increased • Recognise when a contextual problem is following this addition structure • Represent the set from a contextual problem using manipulatives • Use the vocabulary of addition when increasing the set	• When counting on from a given number, includes the given number in their counting • Can count on but does not understand how to apply this to addition • Counts on in ones but does not use this as a method • Can count on but does not understand how to apply this to addition or the combining of two or more groups of objects, resorting to counting all	• Recognise that one more is an increase in the number sequence • Always use a context for augmentation using 'first', 'then' and 'now' as prompts • Use props to enhance number stories • If the total is below 4, allow children to subitize. There is no need for them to count • Encourage children to count all of the objects to find the total, if

	• Find the new total quantity of the sets by using strategy • Know the group is now a larger quantity than the original group • Begin to use marks to record their understanding of augmentation		it is 5 or above • When children are ready, promote the idea of counting on from the existing set. Support this idea with the use of familiar structured representations. Objects within the existing set could be covered or replaced with numerals • Vary types of questions to include missing number problems. Scaffold this by showing the scenario with props and then covering a set over
<u>Decreasing a Set</u> • Understand that subtraction can be the change of an existing set by decreasing its quantity (Reduction)	• Recognising that if objects are removed from a set, the quantity has decreased • Recognise when a contextual problem is following this subtraction structure • Represent the set from a contextual problem using manipulatives • Use the vocabulary of subtraction when removing from the set • Find the reduced quantity of the set by using a strategy • Begin to use marks to record their understanding of reduction	• When counting back from a given number incudes the given number in their counting • Can count back but does not understand how to apply this to subtraction • Count backs in ones but does not use this to subtract	• Counting back to subtract • Make link between one fewer and subtraction one • Recognising that if objects are removed from an existing set the quantity changes • Recognise that one fewer is a decrease in the number sequence • Always use a context for reduction using first, then and now as prompts • Use props to enhance number stories • If what is remaining is below 4, allow children subitize, there is

			no need to count • Encourage children to count what is remaining if is if 5 or above • When children are ready promote the idea of counting back, scaffold this using structured frames • Vary types of question to include missing number problems, scaffold this by showing the scenario with props and then covering a set over • Make explicit the link between 1 fewer and subtracting one

Year 1

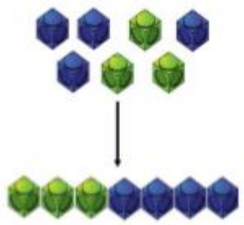
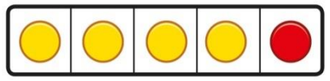
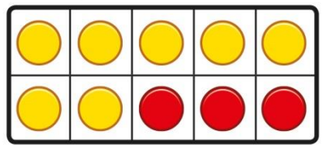
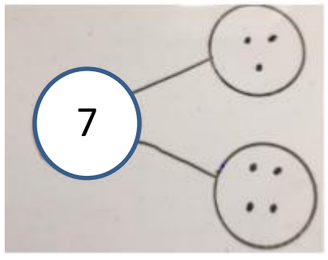
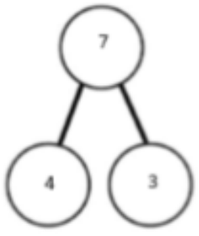
Key Stage 1

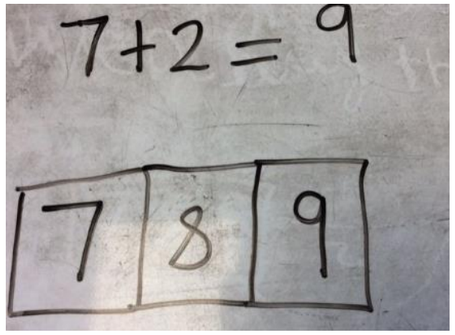
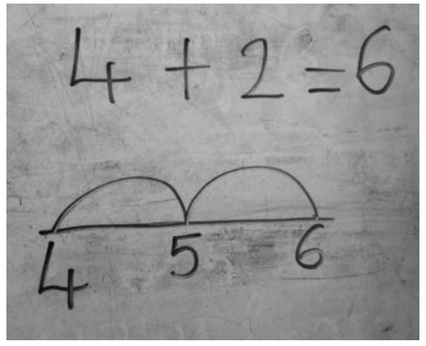
Children in Years 1 and 2 will be given a really solid foundation in the basic building blocks of mental and written arithmetic. Through being taught place value, children will develop an understanding of how numbers work, so that they are confident with 2-digit numbers and beginning to read and say numbers above 100. A CPA approach will be used throughout each unit, ensuring a range of manipulative and representations are used to support children's learning.

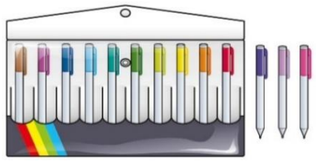
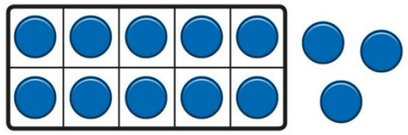
Addition and Subtraction: A focus on number bonds, first via practical hands-on experiences and subsequently using memorisation techniques, enables a good grounding in these crucial facts, and ensures that all children leave Year 2 knowing the pairs of numbers which make all the numbers up to 10 at least. Children will also have experienced and been taught pairs to 20. Children's knowledge of number facts enables them to add several 1-digit numbers, and to add/subtract a 1-digit number to/from a 2-digit number. Another important conceptual tool is the ability to add/subtract 1 or 10, and to understand which digit changes and why. This understanding is extended to enable children to add and subtract multiples of 10 to and from any 2-digit number. The most important application of this knowledge is the ability to add or subtract any pair of 2-digit numbers by counting on or back in 10s and 1s. Children will then extend this knowledge by learning the written method of column addition and subtraction (with regrouping and exchanging) with emphasis on the place value of each digit.

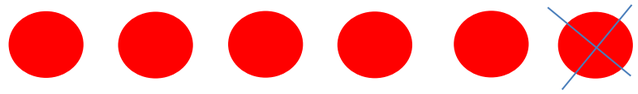
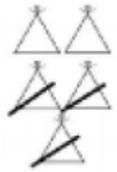
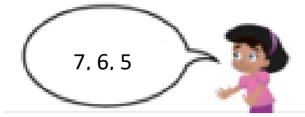
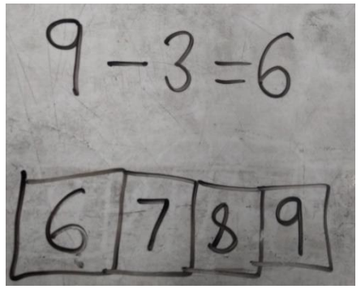
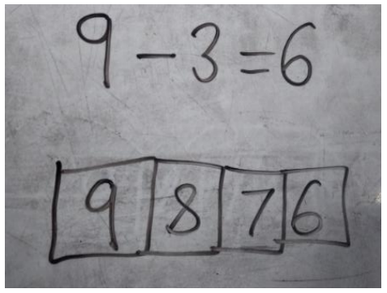
Multiplication and Division: Children will be taught to count in 2s, 3s, 5s and 10s, and will relate this skill to repeated addition. Children will learn the associated ×2, ×3, ×5 and ×10 tables. Engaging in a practical way with the concept of repeated addition and the use of arrays enables children to develop a preliminary understanding of multiplication, and asking them to consider how many groups of a given number make a total will introduce them to the idea of division. Children will also be taught to double and halve numbers, and will thus experience scaling up or down as a further aspect of multiplication and division.

Fractions: Fractions will be introduced as numbers and as operators, specifically in relation to halves, quarters and thirds. Children will learn how to find halves and quarters of shapes and amounts, linking to their learning of division, using the same skills and methods.

	National Curriculum Objectives	Mental Calculation	Written Calculation
	Read, write and interpret mathematical statements involving addition (+), subtraction (–) and equals (=) signs Represent and use number bonds and related subtraction facts within 20 Add one-digit and two-digit numbers to 20, including zero Solve one-step problems that involve addition, using concrete objects and pictorial representations, and missing number problems such as 7 = 4 + ☐	Place the larger number in your head and count on the smaller number to find your answer. 9 + 4 = 13 E.g. If I am at 9, how many more do I need to make 13. How many more do I add on now? Learn number bonds to 10	**<u>Combining two parts to make a whole- (Including number bonds)</u>** **<u>Concrete</u>** (use other resources too e.g. counters, teddy bears, cars and demonstrate on a number frame)  *4 + 3 = 7*  *4 + 1 = 5*  *7 + 3 = 10* **Children to represent the concrete objects using dots or crosses on a part whole model** **<u>Pictorial</u>**  **<u>Abstract</u>**  *4 + 3 = 7 Four is a part, 3 is a part and the whole is seven.*

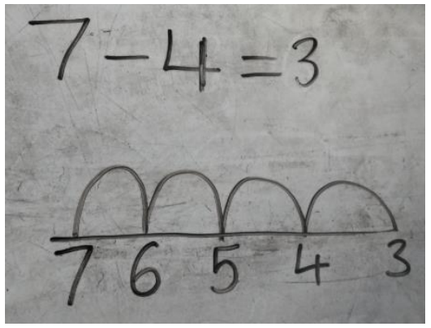
Y1 +			**<u>Starting at the bigger number and counting on</u>** **<u>Concrete</u>** Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer.  *12 + 5 = 17* **<u>Pictorial</u>** Start at the larger number on the number track/ number line and count on in ones  Draw a number track – start at the biggest number and count on  **<u>The abstract number line</u>** *What is 2 more than 4? What is the sum of 2 and 4? What is the total of 4 and 2? 4 + 2 =* 

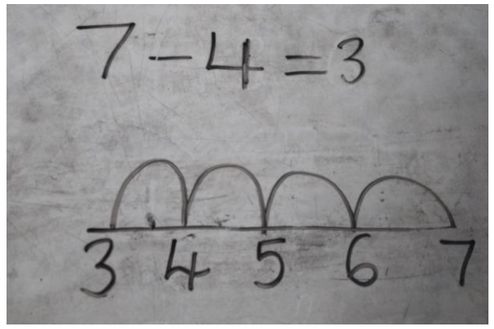
			<u>Understanding teen numbers as a complete 10 and some more</u> **<u>Concrete</u>** Complete a group of 10 objects and count more. *13 is 10 and 3 more. 10 + 3 = 13*  **<u>Pictorial</u>** Use a ten frame to support understanding of a complete 10 for teen numbers.  *13 is 10 and 3 more.* **<u>Abstract</u>** *1 ten and 3 ones equal 13.* *10 + 3 = 13*

Y1 –	Read, write and interpret mathematical statements involving subtraction (-) and equals (=) signs Represent and use number bonds and related subtraction facts within 20 Subtract one-digit and two-digit numbers to 20, including zero Solve one-step problems that involve subtraction, using concrete objects and pictorial representations, and missing number problems such as 7 = – 9.	Place the larger number in your head and count back the smaller number to find your answer. 13 – 4 = 9 E.g. If I am at 13, how many do I need to count back to get to 9? Learn number bonds to 10 and related subtraction sentences	**<u>Counting back and taking away</u>** **<u>Concrete</u>** Children arrange objects and remove to find how many are left.  1 less than 6 is 5. 6 subtract 1 is 5. 6- 1 = 5 Move the beads along the bead string as you count backwards.  13- 4 = 9 **<u>Pictorial</u>** Cross out drawn objects to show what has been taken away.  5- 3 = 2 Children count back to take away and use a number line or number track to support the method.  8 – 3 = 5  Draw a number track – start at the biggest number and count backwards  

The abstract number line

What is 4 less than 7? What is 7 subtract 4? 7 – 4 =

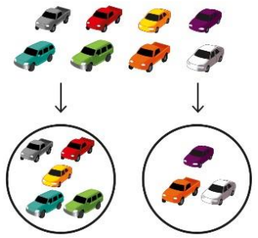




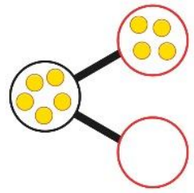
Finding a missing part, given a whole and a part

Concrete
Children separate a whole into parts and understand how one part can be found by subtraction.

8 − 5 =?

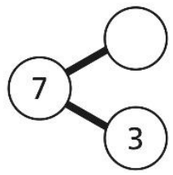


Pictorial
Children represent a whole and a part and understand how to find the missing part by subtraction.

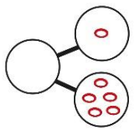


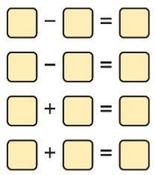
Abstract
Children use a part-whole model to support the subtraction to find a missing part.

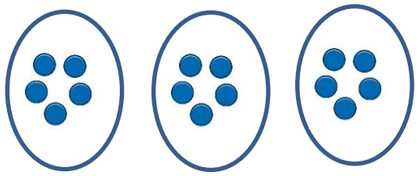
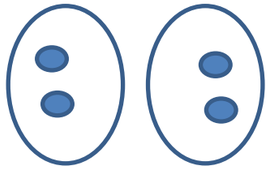
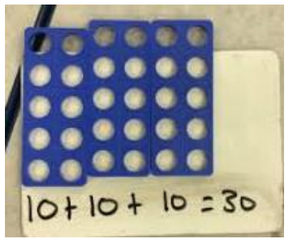
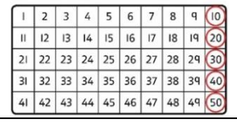
7 − 3 =?

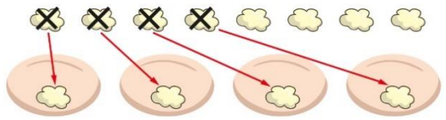
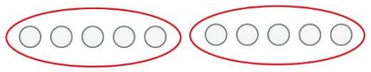


Children develop an understanding of the relationship between addition and subtraction facts in a part-whole model.





Y1 x	Solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher	Count in 2s, 5s and 10s Begin to say what three 5s are by counting in 5s, or what four 2s are by counting in 2s, etc. Double numbers to 10	**<u>Multiplication</u>** **<u>Recognising and making equal groups</u>** **<u>Concrete</u>** Children arrange objects in equal and unequal groups and understand how to recognise whether they are equal.  **<u>Pictorial</u>** Children draw and represent equal groups.  Describe equal groups using words e.g. there are 3 equal groups of 5 NB: This method also to be used when finding doubles of amounts. **2 lots of 4 is the same as double 4**  **<u>Finding the total of equal groups by counting in 2s, 5s and 10s</u>** **<u>Concrete</u>**  *2 4*  *5 10 15*  There are 5 pens in each pack 5…10…15…20…25…30…35…40… **<u>Repeated addition</u>** Use different objects and pictures to add equal groups. Write addition sentences to describe objects and pictures .    Number squares to support counting in 2s, 5s and 10s.

Y1 **÷**	Solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher	Begin to count in 2s, 5s and 10s Find half of even numbers to 12 Find half of even numbers by sharing	**<u>Division</u>** **<u>Grouping</u>** **<u>Concrete</u>** Learn to make equal groups from a whole and find how many equal groups of a certain size can be made. Sort a whole set people and objects into equal groups.  ***There are 10 children altogether.*** ***There are 2 in each group.*** ***There are 5 groups.*** **<u>Pictorial</u>** Represent a whole and work out how many equal groups.  There are 10 in total. There are 5 in each group. There are 2 groups. **<u>Sharing</u>** Share a set of objects into equal parts and work out how many are in each part.  Sketch or draw to represent sharing into equal parts. This may be related to fractions.  10 shared into 2 equal groups gives 5 in each group.  $\frac{1}{2}$ of 10 is 5 NB: This method also to be used when finding fractions of amounts.

Year 2

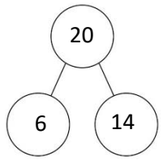
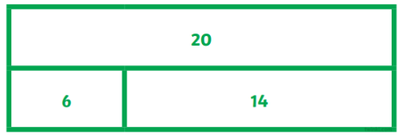
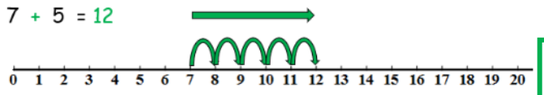
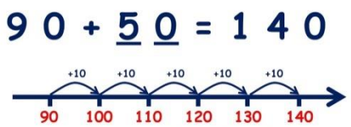
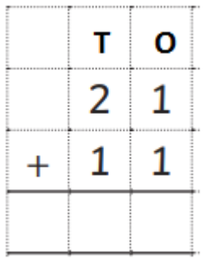
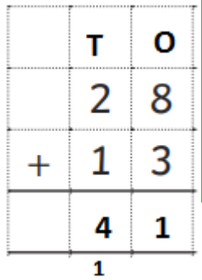
Key Stage 1

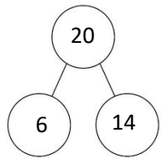
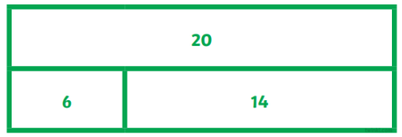
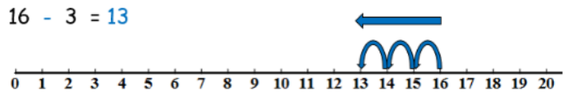
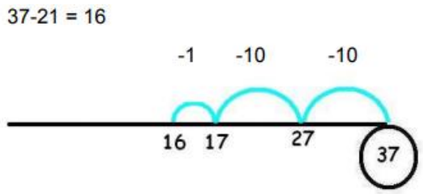
Children in Years 1 and 2 will be given a really solid foundation in the basic building blocks of mental and written arithmetic. Through being taught place value, children will develop an understanding of how numbers work, so that they are confident with 2-digit numbers and beginning to read and say numbers above 100. A CPA approach will be used throughout each unit, ensuring a range of manipulative and representations are used to support children's learning.

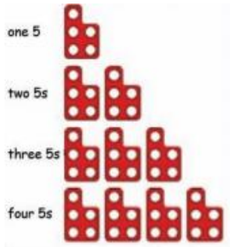
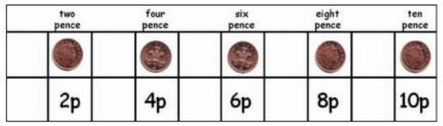
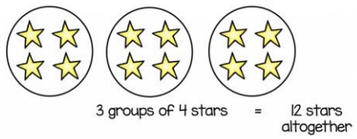
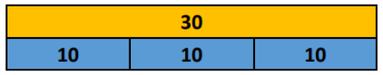
Addition and Subtraction: A focus on number bonds, first via practical hands-on experiences and subsequently using memorisation techniques, enables a good grounding in these crucial facts, and ensures that all children leave Year 2 knowing the pairs of numbers which make all the numbers up to 10 at least. Children will also have experienced and been taught pairs to 20. Children's knowledge of number facts enables them to add several 1-digit numbers, and to add/subtract a 1-digit number to/from a 2-digit number. Another important conceptual tool is the ability to add/subtract 1 or 10, and to understand which digit changes and why. This understanding is extended to enable children to add and subtract multiples of 10 to and from any 2-digit number. The most important application of this knowledge is the ability to add or subtract any pair of 2-digit numbers by counting on or back in 10s and 1s. Children will then extend this knowledge by learning the written method of column addition and subtraction (with regrouping and exchanging) with emphasis on the place value of each digit.

Multiplication and Division: Children will be taught to count in 2s, 3s, 5s and 10s, and will relate this skill to repeated addition. Children will learn the associated ×2, ×3, ×5 and ×10 tables. Engaging in a practical way with the concept of repeated addition and the use of arrays enables children to develop a preliminary understanding of multiplication, and asking them to consider how many groups of a given number make a total will introduce them to the idea of division. Children will also be taught to double and halve numbers, and will thus experience scaling up or down as a further aspect of multiplication and division.

Fractions: Fractions will be introduced as numbers and as operators, specifically in relation to halves, quarters and thirds. Children will learn how to find halves and quarters of shapes and amounts, linking to their learning of division, using the same skills and methods.

	National Curriculum Objectives	Mental Calculation	Written Calculation
Y2 +	**Add numbers using concrete objects, pictorial representations, and mentally, including:** • **a two digit number and ones;** • **a two digit number and tens;** • **two two-digit numbers; three one-digit numbers.**	Use place value knowledge to find one more and ten more than any 2-digit number up to 100. Count on in multiples of 10, 5, 2 and 3 and in tens from any number. Use patterns of known facts. **Eg. 7 + 2 = 9 so 27 + 2 = 29** Use number bonds to 10 knowledge when adding three or more single digit numbers. **Eg. 8 + 4 + 2 as 10 + 4**	***Part Whole Models and Bar Models to visualise number bonds and simple addition calculations***  6 + 14 = 20 14 + 6 = 20 20 = 14+6 20 = 14 + 6  ***Number lines for 2-digit add 1 digit and for adding multiples of 10***  NB: When initially introducing written strategies, simpler calculations (which would normally be solved mentally) may be used to demonstrate the method before progressing onto more challenging calculations.  ***Column Addition for adding two 2-digit numbers***   NB: Emphasis to be made on the place value of each digit so children do not think it is 2 +1. Ask questions such as 'What is the value of 2 in this calculation?', 'Can you show me this number partitioned?'

Y2 –	**Subtract numbers using concrete objects, pictorial representations, and mentally, including:** **• a two digit number subtract ones;** **• a two digit number subtract tens;** **• two-digit numbers subtract two-digit.**	Use place value knowledge to find one more and ten more than any 2-digit number up to 100. Count back in multiples of ten from 100. Using concrete initially, moving on to 100 square and then mentally. Use patterns of known facts. **Eg. 5 – 2 = 3 so 25 – 2 = 23**	***Continue to use part whole models and bar models to represent related addition and subtraction facts.***  6 + 14 = 20 14 + 6 = 20 **So…** 20 – 14 = 6 20 – 6 = 14  ***Number lines for 2-digit minus 1 digit and for subtracting multiples of 10***   ***Column subtraction for two 2-digit number, with regrouping*** 2 ~~3~~ ¹3 - 1 5 1 8 NB: Emphasis to be made on the place value of each digit and when introduced to the method children should be shown it using Numicon or Base 10 to model the exchanging of tens and ones.

Y2 x	**Recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers.** **Calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (×), division (÷) and equals (=) signs.** **Show that the multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot.** **Solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods and multiplication and division facts, including problems in context.**	Children to use a range of vocabulary to describe multiplication and use a variety of practical resources to explain multiplication. Use songs to aid children's initial ability to recite counting in 10s, 2s, 5s and 3s (in that order). Rote learn times tables to increase children's rapid recall.	***Count in 2s, 5s, 10s and 3s using variety of concrete and pictorial representations***    ***Arrays for multiplication*** 3 x 4 = 12  Show multiplication as repeated addition. 3 +3+3+3=12 4 x 3 = 12  Rotate arrays to show that multiplication of two numbers can be done in any order **(commutative law)** ***Use other pictorial representations to help children visualise the concept of multiplication.***  

Y2 ÷	**<u>Recall and use division facts for the 2, 5 and 10 multiplication tables.</u>** **<u>Solve problems involving division, using materials, arrays, repeated subtraction, mental methods, and multiplication and division facts, including problems in contexts.</u>** **<u>Calculate mathematical statements for division within the multiplication tables and write them using the division (÷) and equals (=) signs.</u>** **<u>Show that division of one number by another is not commutative [i.e. can be done in any order].</u>**	Children to use a range of vocabulary to describe division and use a variety of practical resources to explain multiplication. Use their multiplication knowledge to derive known division facts. **Eg. 5 x 10 = 50 so 50 ÷ 10 = 5**	***<u>Make links to multiplication by continuing to use arrays to support division.</u>***  NB: Continue to reinforce sharing and grouping in a practical context. How many groups of 3? How many groups of 5? 15 shared between 3 people is....? 15 shared between 5 people is....? 15 divided by 3 = 5 15 divided by 5 = 3 15 ÷ 3 = 5 15 ÷ 5 = 3 ***<u>Use pictorial representations to share into equal groups</u>*** 12 ÷ 3 = 4  NB: This method also to be used when finding fractions of amounts. $\frac{1}{2}$ of 8 is 4 

Year 3

LOWER KEY STAGE 2

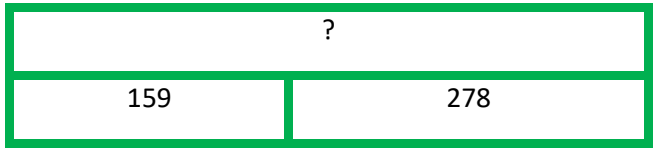
The principal focus of mathematics teaching in lower key stage 2 is to ensure that pupils become increasingly fluent with whole numbers and the four operations, including number facts and the concept of place value. This should ensure that pupils develop efficient written and mental methods and perform calculations accurately with increasingly large whole numbers.

Addition and subtraction: Children are taught to use place value and number facts to add and subtract numbers mentally and they will develop a range of strategies to enable them to become less reliant on the 'counting in 1s' or fingers-based methods of Key Stage 1. In particular, children will learn to add and subtract multiples and near multiples of 10, 100 and 1000 (year 4) and will become fluent in complementary addition as an accurate means of achieving fast and accurate answers to 3-digit subtractions. Standard written methods for adding larger numbers are taught, learned and consolidated, and written column subtraction is also introduced.

Multiplication and division: This key stage is also the period during which all the multiplication and division facts are thoroughly memorised, including all facts up to 12 × 12. Efficient written methods for multiplying or dividing a 2-digit or 3-digit number by a 1-digit number are taught, as are mental strategies for multiplication or division with large but 'friendly' numbers, e.g. when dividing by 5 or multiplying by 20.

Fractions and decimals: Children will develop their understanding of fractions, learning to simplify fractions and find equivalents as well as finding fractions of amounts and quantities. The concept of a decimal number is introduced and children consolidate a firm understanding of 1-place decimals, dividing whole numbers by 10 and 100 and seeing the effect on the digits.

	National Curriculum Objectives	Mental calculation	Written calculation
Y3 +	**<u>Add and Subtract numbers mentally, including:</u>** 1. a three-digit number and 1s 2. a three-digit number and 10s 3. a three-digit number and 100s **<u>Add and subtract numbers with up to 3 digits, using formal written methods of</u>**	Use place value knowledge to add a 3-digit number and ones, tens and hundreds up to 1000. Place value grids and counters are used to help children visualise and understand what they are doing mentally. Children are encouraged to use the basic number facts they know to help them.	**<u>Continue to use part whole models and bar models</u>** **<u>Use to represent related addition and subtraction facts.</u>** 759 444 315 444 + 315 = 759 315 + 444 = 759 **So…** 759 – 444 = 315 759 – 315 = 444 759 444 | 315 **<u>Use to help solve missing number problems/ inverse. Use to check answers to a calculation.</u>** We know that 159 + 278= ? We can help visualise this problem by putting it into a bar model (or part whole model), now we know we need to add them together

	columnar addition and subtraction **Estimate the answer to a calculation and use inverse operations to check answers**	For example: **Adding ones:** **5 + 3 = 8** so, 34**5** + **3** = 34**8** **6 + 4 = 10** so, 45**6** + **4** = 460 **Adding tens:** **70 + 20 = 90** so, 8**7**6 + **2**0 = 8**9**6 Where numbers bridge over 100, children are encouraged to look at the hundreds and tens as a 2-digit number: **89**1 + 10 = **90**1 **Adding hundreds:** **400 + 300= 700** so, **4**72 + **3**00= **7**72 Relate number bonds to 10 to number bonds to 100 and 1000 (e.g. 3 + 7 = 10 so 30 + 70 = 100 therefore 300 + 700 = 1000 and be able to recall them.	 ? 159 278 **Column addition for up to two 3-digit number, with 1 or more regrouping** Use of (compact) column addition with up to two 3-digit numbers (may also do 3 digit number + 2 digit number, or three 3 digit numbers added together etc). May have no regrouping, one regroup or multiple regroups. **Regroup once** Starting with the ones, add each column in turn. When adding 4 ones + 8 ones = 12 = 1 ten and 2 ones. Place 1 ten <u>under</u> the equal sign on the ten column and the 2 ones in the answer ('hang it on the washing line') **Regroup multiple times** Starting with the ones, add each column in turn. Regroup tens and hundreds as required ('hang it on the washing line')

			NB: Children to understand commutative law. Numbers can be added in any order and it will not effect the answer. NB: Emphasis to be made on the place value of each digit so children do not think it is 8 - 7. Ask questions such as 'What is the value of 8 in this calculation?', 'Can you show me this number partitioned?' **Estimate the answer to a calculation** Children to look for the nearest multiple of 10 or 100 and add the 2 numbers together to get an estimate. **51 + 29 = ☐ 50 + 30 = 80** **204 + 198 = ☐ 200 + 200 = 400**
	<u>Add and subtract</u>		**<u>Add and subtract fractions with the same denominator</u>**

	fractions with the same denominator within one whole **Add and subtract amounts of money to give change, using both £ and p in practical contexts**		1. Children use practical equipment and pictorial representations to add two or more fractions with the same denominator where the total is less than 1. 2. Children understand that we only add the numerators and the denominators stay the same. **Adding amounts of money** Children add two amounts of money using pictorial representations to support them. They are encouraged to add the pounds first and then add the pence. Children then exchange the pence for pounds to complete their calculations. £2 and 35p + £1 and 75 p. There is £3 and 110p. Altogether there is £1 and10p. £5 and 30p + £3 and 75p. There is £8 and 105p. Altogether there is £9 and 5p.

Y3 –	**<u>Add and subtract numbers mentally, including:</u>** **<u>Add and subtract numbers with up to 3 digits, using formal written methods of columnar addition and subtraction</u>**	Use place value knowledge to a subtract a 3-digit number and ones, tens and hundreds up to 1000. Place value grids and counters are used to help children visualise and understand what they are doing mentally. Children are encouraged to use the basic number facts they know to help them. For example: **<u>Subtracting ones:</u>** **5 - 3 = 2** so, 34<u>**5**</u> - <u>**3**</u> = 34**2** **<u>Subtracting tens:</u>** **70 - 20 = 50** so, 8<u>**7**</u>6 - <u>**2**</u>0 = 8<u>**5**</u>6 Where numbers bridge over 100,	**<u>Continue to use part whole models and bar models</u>** **<u>Use to represent related addition and subtraction facts.</u>** 567 233 334 233 + 334 = 567 334 + 233 = 567 **So…** 567 – 334 = 233 567 – 233 = 334 567 334 223 **<u>Use to help solve missing number problems and represent inverse</u>**. We know that 781 - ? = 365 We can help visualise this problem by putting it into a bar model (or part whole model) like on the right. Now we can see the other subtraction we need to do.

children are encouraged to look at the hundreds and tens as a 2-digit number:

<u>80</u>1 - 10 = **<u>79</u>**1

<u>Subtracting hundreds:</u>

400 – 300 = 100 so,

<u>4</u>72 - **<u>3</u>**00= **<u>1</u>**72

881	
?	365

881

? 365

<u>Column subtraction for up to two 3-digit number, with 1 or more exchange</u>

Use of (compact) column subtraction with up to two 3-digit numbers (may also do 3-digit number – 2 digit number etc). May have no exchanging, one exchange or multiple exchanging.

One exchange

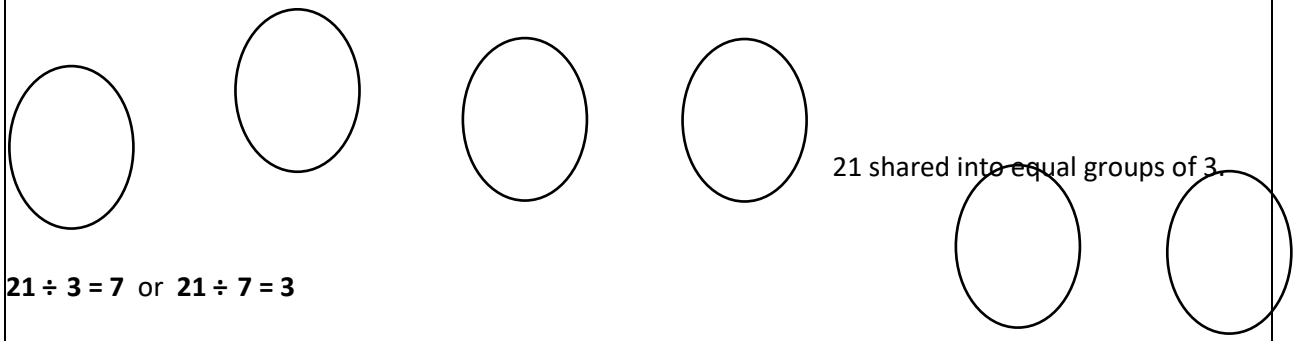
Starting with the ones, subtract each column in turn. When subtracting 0 ones from 5 ones, exchange 1 ten from the tens column to make 1 ten and 4 ones (14). Change the 4 tens into 3 tens.

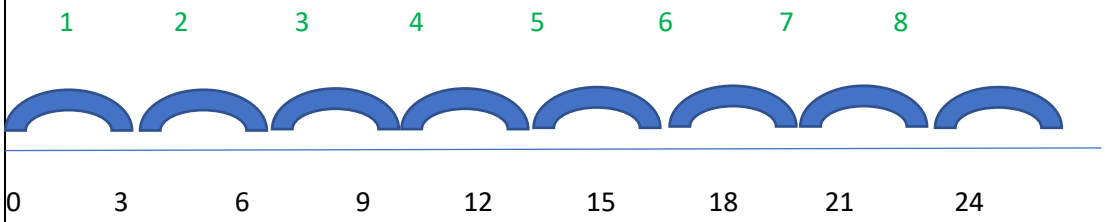
	<u>Estimate the answer to a calculation and use inverse operations to check answers</u> **<u>Add and subtract fractions with the same denominator within one whole</u>**		NB: Emphasis to be made on the place value of each digit so children do not think it is 2 - 1. Ask questions such as 'What is the value of 2 in this calculation?', 'Can you show me this number **Multiple exchanging** Starting with the ones, subtract each column in turn. Exchange in the tens / hundreds as required **Estimation:** Children to look for the nearest multiple of 10 or 100 and subtract the 2 numbers to get an estimate. **59 – 31 = ☐ 60 – 30 = 30** **598 - 203 = ☐ 600 - 200 = 400** **<u>Add and subtract fractions with the same denominator</u>** Children use practical equipment and pictorial representations to subtract fractions with the same denominator within one whole.

	Add and subtract amounts of money to give change, using both £ and p in practical contexts		Children understand that we only subtract the numerators and the denominators stay the same. **Subtracting amounts of money** Children use different methods to subtract money. They will see examples where they can physically remove the coins, and examples where they will need to use their knowledge of converting money to exchange £1 for 100 pence. Children also use number lines to count on or back to calculate the difference between two amounts. Alex has £3 and 50p. She gives £2 and 10p to her sister. How much money does she have left? £3 − £2 = £1. 50p − 10p = 40p. Alex has £1 and 40p remaining. Tommy has £1 and 72p. Rosie has £2 How much more money does Rosie have than Tommy?

			Rosie has 28p more than Tommy
Y3 ×	**Recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables** **Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods**	Know by heart all the multiplication facts in the ×3, ×4 and ×8 tables Recognise that multiplication is commutative 3 x 5 = 15 5 x 3 = 15 Multiplying by 1 Know that any number x by 1 = itself **For example: 8 x 1 = 8** Multiplying by 0 Know that any number x by 0= 0	Understanding multiplication as equal groups of and that multiplication is commutative. There are 3 equal groups of 4. 3 x 4 = 12 or 4 x 3 = 12 There are 5 equal groups of 8. 5 x 8 = 40 or 8 x 5 = 40 Using known multiplication facts and partitioning to answer 2 digit by 1digit calculations : 3 2 x 3 = / \ 30 2 30 x 3 = 90 (3 x 3) 2 x 3 = 6 30 + 6 = 36 **Formal written method: 2 digit numbers by 1 digit number (2, 3, 4, 5 and 8 times tables)**

NB: Reinforce division facts as inverse of multiplication throughout teaching.

			<u>No regrouping</u> NB: Emphasis to be made on the place value of each digit so children do not think it is 2 x 3 . Ask questions such as 'What is the value of 3 in this calculation?', 'Can you show me this number partitioned?' **With Regrouping**
Y3 ÷	**<u>Recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables</u>**	Know by heart all the division facts derived from the ×2, ×3, ×4, ×5, ×8 and ×10 tables. Recognise that division is not commutative Use place value and number facts in mental division Check that Children can halve even numbers to 100, halve odd numbers to 20	**<u>Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods</u>** To understand division as equal groupings:  **21 ÷ 3 = 7** or **21 ÷ 7 = 3** **<u>Chunking on a number line: (numbers that will divide equally by 2, 3, 4, and 8)</u>** Using a number line to count from zero in the multiple until you get to the required amount. Count the

		NB: Reinforce multiplication facts as inverse of division throughout teaching	number of jumps made to get the answer. **24 ÷ 3 = 8**  **<u>Bus stop method: (2 and 3 digit, multiples of 2, 3, 4, 5 and 8- no remainders)</u>** **No regrouping** NB: Emphasis to be made on the place value of each digit so children do not think it is 3 ÷ 6. Ask questions such as 'What is the value of 6 in this calculation?', 'Can you **With regrouping**

			Diagrams to help: 1 equal group of 8 and 1 remaining 1 equal group of 4 and 2 remaining

Year 4

Lower Key Stage 2

The principal focus of mathematics teaching in lower key stage 2 is to ensure that pupils become increasingly fluent with whole numbers and the four operations, including number facts and the concept of place value. This should ensure that pupils develop efficient written and mental methods and perform calculations accurately with increasingly large whole numbers.

Addition and subtraction: Children are taught to use place value and number facts to add and subtract numbers mentally and they will develop a range of strategies to enable them to become less reliant on the 'counting in 1s' or fingers-based methods of Key Stage 1. In particular, children will learn to add and subtract multiples and near multiples of 10, 100 and 1000 (year 4) and will become fluent in complementary addition as an accurate means of achieving fast and accurate answers to 3-digit subtractions. Standard written methods for adding larger numbers are taught, learned and consolidated, and written column subtraction is also introduced.

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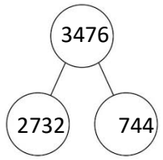
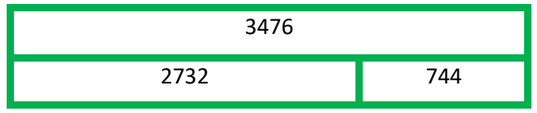
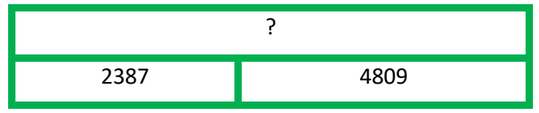
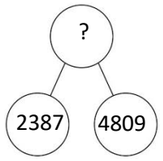
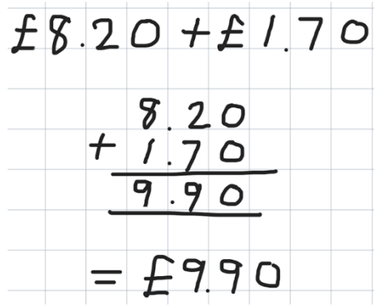
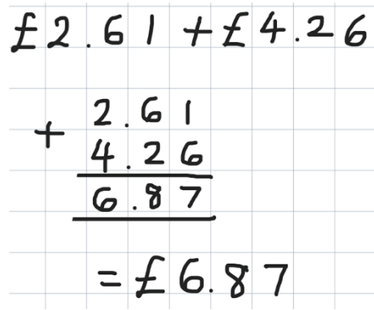
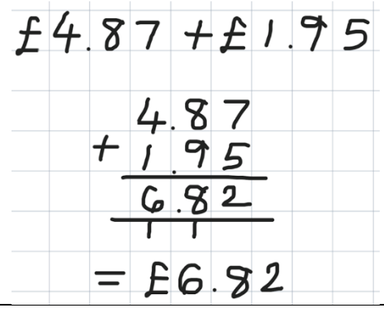
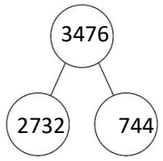
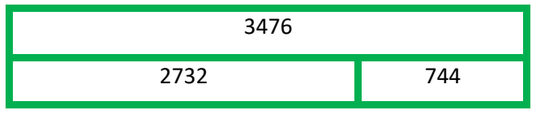
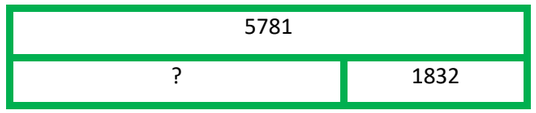
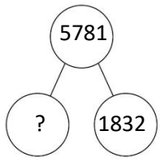
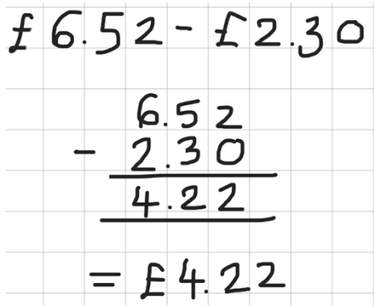
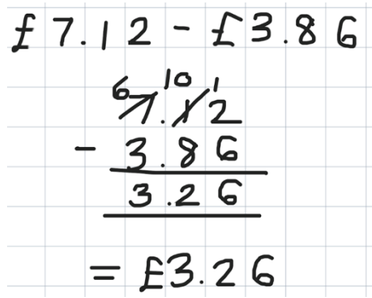
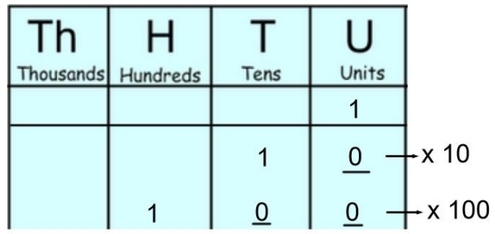
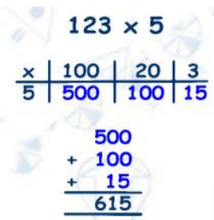
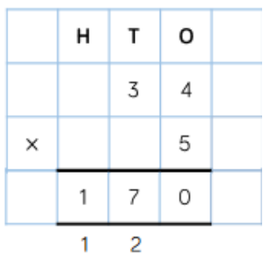
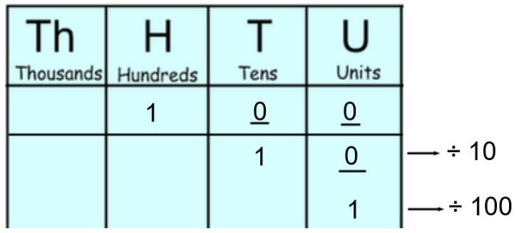
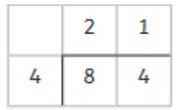
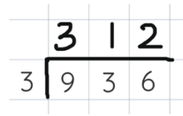
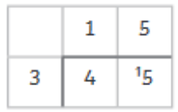
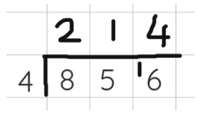
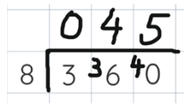
	National Curriculum Objectives	Mental Calculation	Written Calculation
Y4 +	**<u>Add numbers with up to 4 digits using the formal written methods of columnar addition where appropriate</u>** **<u>Estimate and use inverse operations to check answers to a calculation</u>** **<u>Solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why</u>**	Simple mental addition to ensure no errors with column addition. Use of place value to find 10, 100 or 1000 more. Use of place value to find more than a given number and including in negative numbers. **For example: Find 3 more than -8.** Use number line initially, then jottings and then mentally Relate number bonds to 10 to number bonds to 100 and 1000 (e.g. 3 + 7 = 10 so 30 + 70 = 100 therefore 300 + 700 = 1000 and be able to recall them. NB: Emphasis to be made on the place value of each digit and when introduced in Y4 (already done version of this in Y2 and Y3) to the method children should be shown it with counters and place value grid on the IWB to model regrouping. If need practical apparatus - use Numicon or Base 10 to model the regrouping.	**<u>Continue to use part whole models and bar models</u>** **<u>Use to represent related addition and subtraction facts.</u>**  2732 + 744 = 3476 744 + 2732 = 3476 **So…** 3476 – 2732 = 744 3476 – 744 = 2732  **<u>Use to help solve missing number problems/ inverse. Use to check answers to a calculation.</u>** We know that 2387 + 4809 = ? We can help visualise this problem by putting it into a bar model (or part whole model) like on the right, now we know we need to add them together. We can do 2387 + 4809 to find our missing number (=7196). We can now do 7196 – 2387 to check. If we get 4809 we are correct.   **<u>Column addition for up to two 4-digit number, with 1 or more regrouping</u>** Use of (compact) column addition with up to two 4-digit numbers (may also do 4 digit number + 3 digit number, or three 4 digit numbers added together etc). May have no regrouping, one regroup or multiple regroups. **Regroup once** 5162 +3497 8659 1 Starting with the ones, add each column in turn. When adding 6 tens + 9 tens = 15 tens = 1 hundred = 5 tens. Place 1 hundred <u>under</u> the equal sign on the hundred column and the 5 tens in the answer ('hang it on the washing line') **Regroup multiple times** 5864 +3497 9361 1 1 1 Starting with the ones, add each column in turn. Regroup tens, hundreds and/or thousands as required ('hang it on the washing line')

Y4 + continued	(See above)	(See above)	**<u>Column addition for decimals</u>** Use of (compact) column addition for numbers with the same amount of decimal places **For example**: when solving addition problem with a money context that goes into the decimal system with tenths and hundredths For two amounts with same number of decimal places (only tenths):  For two amounts with same number of decimal places (tenths and hundredths):  For two amounts with same number of decimal places (tenths & hundredths) and require regrouping: 

Y4 –	**<u>Subtract numbers with up to 4 digits using the formal written methods of columnar subtraction where appropriate</u>** **<u>Estimate and use inverse operations to check answers to a calculation</u>** **<u>Solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why</u>**	Simple mental subtraction to ensure no errors with column subtraction. Use of place value to find 10, 100 or 1000 less. Use of place value to find less than a given number and going into negative numbers. **For example: Find 7 less than 2.** Use number line initially, then jottings and then mentally	**<u>Continue to use part whole models and bar models</u>** **<u>Use to represent related addition and subtraction facts.</u>**  2732 + 744 = 3476 744 + 2732 = 3476 **So…** 3476 – 2732 = 744 3476 – 744 = 2732  **<u>Use to help solve missing number problems and represent inverse</u>**. We know that 5781 - ? = 1832 We can help visualise this problem by putting it into a bar model (or part whole model) like on the right. Now we can see the other subtraction we need to do. We now know we can do 5781 – 1832 to find our missing number.   **<u>Column subtraction for up to two 4-digit number, with 1 or more exchange</u>** Use of (compact) column subtraction with up to two 4-digit numbers (may also do 4 digit number – 3 digit number etc). May have no exchanging, one exchange or multiple exchanging. One exchange 6 1 5749 - 3471 2278 Starting with the ones, subtract each column in turn. When subtracting 4 tens -7 tens, exchange 1 hundred to make: 14 tens – 7 tens = 7 tens Multiple exchanges 6 13 1 5742 - 3476 2266 Starting with the ones, subtract each column in turn. Exchange tens, hundreds and/ or thousands as required. NB: Emphasis to be made on the place value of each digit and when introduced in Y4 (already done version of this in Y2 and Y3) to the method children should be shown it with counters and place value grid on the IWB to model exchanging. If need practical apparatus - use Numicon or Base 10 to model the exchanging.

Y4 – continued	(See above)	(See above)	**<u>Column Subtraction</u>** **<u>Column subtraction for decimals</u>** Use of (compact) column subtraction for numbers with the same amount of decimal places **For example**: when solving subtraction problem with a money context that goes into the decimal system with tenths and hundredths. For two amounts with same number of decimal places:  For two amounts with same number of decimal places and require exchanging: 

Y4 x	**Recall multiplication facts for multiplication tables up to 12 × 12** **Use place value, known and derived facts to multiply mentally, including: multiplying by 0 and 1; multiplying together 3 numbers** **Recognise and use factor pairs and commutativity in mental calculations** **Multiply two-digit and three-digit numbers by a one-digit number using formal written layout** **Solve problems involving multiplying and adding, including using the distributive law to multiply two-digit numbers by 1 digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects**	(not necessarily in this order) Learn shortcuts for mental multiplication **For example:** • to x4 you x2 and x2 again • to x5 you x10 and divide by 2 • to x20 you x2 and x10 Use known multiplication facts to mentally solve other multiplications **For example:** if you know 8x3=24 you know… • 8x30=240 • 80x3=240 • 80x30=2400 Multiplication is commutative Factor pairs of numbers Fact families – using known fact to find the others **For example:** if you know 9x4=36 then you know 4x9=36 and 36÷9=4 and 36÷4=9 Multiplying by 1 Know that any number x by 1 = itself **For example: 81 x 1 = 81** Multiplying by 0 Know that any number x by 0= 0 **For example: 72 x 0 = 0** Multiplying 3 numbers together and shortcuts to take **For example:** 8 x 7 x 2 First solve 8 x 7 = 56 (as it's the trickier one) Then 56 x 2 = 112 (as doubling is easier)	**Multiplying a number by 10 and 100** Use of place value grids Moving 1 place to the left for x10 or 2 places to the left for x100 or 3 places to the left for x1000 NB: Start with counters then write digits in.  Once children understand the place value reasons behind this they can use shortcuts of putting zeros onto the end (making it more of a mental calculation) **For example**: 78 x 10. Multiplying by 10 → 10 has 1 zero so I need to put 1 zero on the end of my number → 780 **For example**: 6 x 100. Multiplying by 100 → 100 has 2 zeroes so I need to put 2 zeroes on the end of my numbers → 600 **For example**: 52 x 100. Multiplying by 100 → 100 has 2 zeroes so I need to put 2 zeroes on the end of my numbers → 5200 **Children to learn both of the following methods and choose what they prefer to use (guided towards choosing column method.)** NB: If children struggled, can use either of these methods with counters & PV grid. **Using grid method for multiplication**  Multiplying 2 and 3 digit numbers x 1 digit numbers using grid method. • First – partition the number into its (hundreds,) tens and ones. • Draw grid and set out partitioned numbers into the grid. • Multiply each partitioned number along the top by the 1 digit number, fill in the answer • Line up all the parts of the answer and complete a column addition • Now you have the final answer **Using column method for multiplication**  Multiplying 2 and 3 digit numbers x 1 digit numbers using column multiplication method • First set out the numbers in a column method ensuring HTO are accurately lined up • Start by multiplying the ones by the x number • Record the answer under the line in the correct column • Work through the tens and then hundreds. • If the digits are larger than 9 they need to be regrouping into the next place value column as the children are familiar in doing with column addition.

Y4 **÷**	**(Multiplication & Division)** **Recall division facts for multiplication tables up to 12 × 12** **Use place value, known and derived facts to divide mentally, including dividing by 1** **Recognise and use factor pairs and commutativity in mental calculations** **(Fractions & Decimals)** **Find the effect of dividing a one- or two-digit number by 10 and 100, identifying the value of the digits in the answer as ones, tenths and hundredths**	Use their multiplication knowledge to divide mentally. **Eg. 8 x 9 = 72 so 72 ÷ 8 = 9** Use inverse of factors and factor pairs. **Eg. Know that factors of 36 are 1 and 36, 2 and 18, 3 and 12, 4 and 9, and 6. Therefore we know that 36 ÷ 3 = 12.** Know that division is not commutative. Fact families – using known fact to find the others **Eg. if you know 9x4=36 then you know 4x9=36 and 36÷9=4 and 36÷4=9** Know that any number ÷ by 1 = itself **Eg. 81 ÷ 1 = 81**	***Dividing a number by 10 and 100*** Use of place value grids Moving 1 place to the right for ÷10 or 2 places to the right for ÷100 NB: Start with counters then write digits in.  ***Bus stop Division*** Bus stop division for 2 or 3 digit numbers divided by 1 digit number (no remainders) Start with numbers that fully divide (no regrouping required) - with 2 digit  or 3 digit  Then move onto some regrouping across - with 2 digit  or 3 digit & one regroup  or 3 digit & one regroup 

Year 5

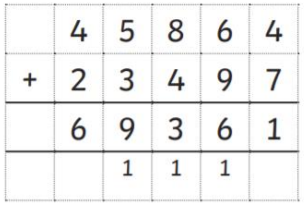
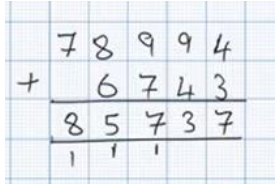
UPPER KEY STAGE 2

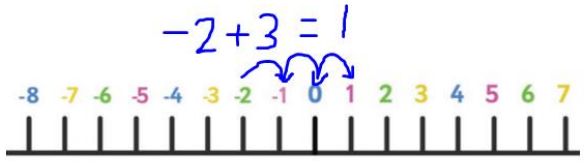
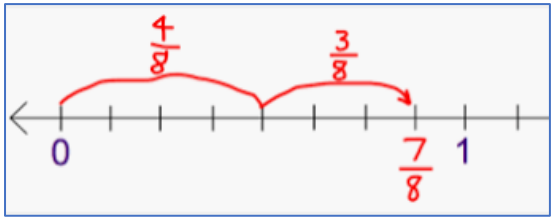
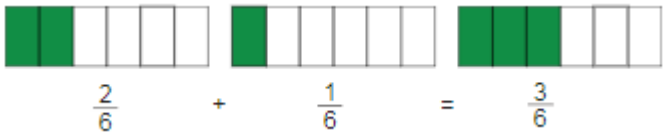
Children move on from dealing mainly with whole numbers to performing arithmetic operations with both decimals and fractions. By the end of year 6, pupils should be fluent in written methods for all four operations, including long multiplication and division, and in working with fractions, decimals and percentages.

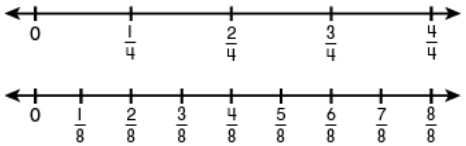
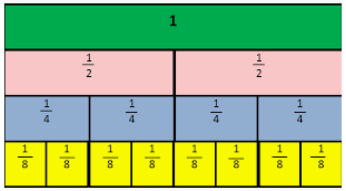
Addition and subtraction: Children will consolidate their use of written procedures in adding and subtracting whole numbers with up to 6 digits and also decimal numbers with up to 3 decimal places. Mental strategies for adding and subtracting increasingly large numbers will also be taught. These will draw upon children's robust understanding of place value and knowledge of number facts. Negative numbers will be added and subtracted.

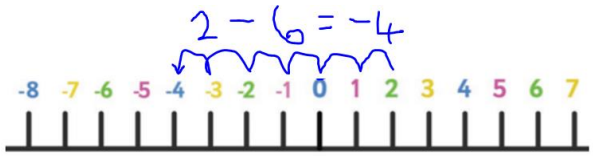
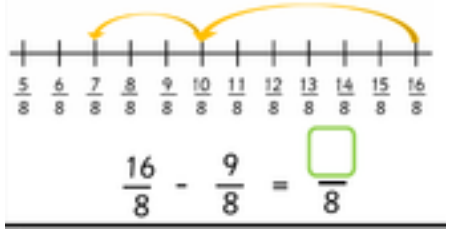
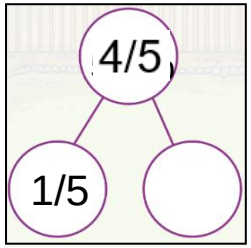
Multiplication and division: Efficient and flexible strategies for mental multiplication and division are taught and practised, so that children can perform appropriate calculations even when the numbers are large, such as 40 000 × 6 or 40 000 ÷ 8. In addition, it is in Years 5 and 6 that children extend their knowledge and confidence in using written algorithms for multiplication and division.

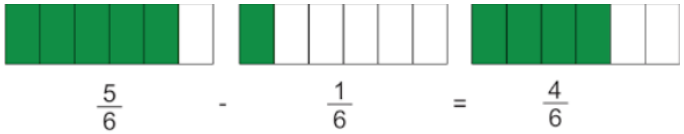
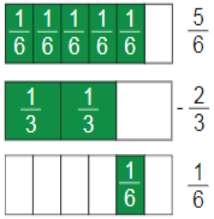
Fractions, decimals and percentages: Fractions and decimals are also added, subtracted, divided and multiplied, within the bounds of children's understanding of these more complicated numbers. Children will also calculate percentages and ratios.

	National Curriculum Objectives	Mental Calculation	Written Calculation- including concrete, pictorial and abstract methods
Y5 +	**<u>Add whole numbers with more than 4 digits, including using formal written methods</u>** **<u>Add numbers mentally with increasingly large numbers</u>**	Simple mental addition to ensure no errors with column addition.	**<u>Column method for addition including regrouping.</u>** Children will be working with place value of numbers up to 1,000,000 in year 5 and will continue to build upon the column addition skills they have worked on in Y4 by calculating with numbers with more than 4 digits.  Starting with the ones, add each column in turn. Regroup tens, hundreds, thousands, ten thousands as required.  N.B. Children are encouraged to put their regrouped digit wherever they feel suits them best. They are shown different ways and are allowed to choose **<u>Children will also use this method to add numbers that have up to 3 decimal places</u>** N.B. Children are given problems which involve adding numbers with differing place value and involving whole numbers added to numbers with decimal places. We teach children to use place holders to help them to line their digits up with the correct place value.

		Use of place value to find 10, 100, 1,000, 10,000, 100,000 or 1,000,000 more. **Negative numbers** Find 12 more than -8. Children to use a number line to start with and then use counting through 0 to support with this type of calculation e.g. -8 to 0 = 8. 0 + 4 = 4. **Number bonds** Have a focus on quick and accurate recall of number bonds to 100 (in tens and ones) and to 1000 (in hundreds and tens) and be able to apply these to larger numbers e.g. 51+49=100 so 510 +490 =1000 and therefore 5100 + 4900 = 10,000 etc.	 **Adding fractions with the same denominator** Children are taught using a range of different models. They are taught to count in fractions and use number lines to add fractions of the same denominator.  They are also taught addition of fractions using the bar model.  These pictorial representations demonstrate that when adding fractions of the same denominator, only the numerators are added and the denominator stays the same. The children can then use a more abstract method as shown. $\frac{2}{9} + \frac{5}{9} = \frac{7}{9}$
	Add fractions with the same denominator and denominators that are multiples of the same number		

			<u>Adding Fractions with denominators that are multiples of the same number</u>. Children are taught to use their knowledge of equivalent fractions to convert the fractions to the same denominator before adding them. For example, 4/8 + ¼ First they would convert 4/8 to 2/4 at first using pictorial representations (bar model, number line, fraction wall etc) and then the more abstract way of multiplying the numerator and denominator by the same number.    Then they would add the numerators together 2/4 + ¼ = ¾
Y5 –	**<u>Subtract numbers mentally with increasingly large numbers</u>**	**<u>Number bonds</u>** Have a focus on quick and accurate recall of number bonds to 100 (in tens and ones) and to 1000 (in hundreds and tens) and be able to apply these to larger numbers e.g. 100 – 49 = 51 so 1,000 – 490 = 510 and therefore 10,000 – 4900 = 5,100 etc. **<u>Compensating and bridging</u>** Children are taught to use rounding to support with the mental calculation of subtracting larger numbers e.g. 4,000 – 1998. Children are to round to the nearest ten, hundred,	

	Subtract whole numbers with more than 4 digits, including using formal written methods **Subtract fractions with the same denominator and denominators that are multiples of the same number**	thousand, ten thousand, hundred thousand dependent on the calculation. 4,000 – 1998 would be 4,000 – 2,000 and then the two would need to be added back on. **Negative numbers** Find 12 less than 8. Children to use a number line to start with and then use counting through 0 to support with this type of calculation e.g. 8 – 8 = 0 and 0 – 4 leftover = -4	  **Column method for subtraction including exchanging.** Children will be working with numbers up to 1,000,000 in year 5 and will continue to build upon the column subtraction skills they have worked on in Y4 by calculating with numbers with more than 4 digits. 3 5 ~~7~~ ~~4~~ ~~2~~ - 3 4 7 6 3 2 2 6 6 Starting with the ones, subtract each column in turn. Exchange tens, hundreds, thousands and/or ten thousands as required. **Subtracting fractions with the same denominator.** Children are taught using a range of different models. They are taught to count in fractions and use number lines to add fractions of the same denominator.   They are also taught subtraction of fractions using the bar model and also represent it using part-whole models.

			 $\frac{5}{6} - \frac{1}{6} = \frac{4}{6}$ These pictorial representations demonstrate that when subtracting fractions of the same denominator, only the numerators are subtracted and the denominator stays the same. The children can then use a more abstract method as shown. $\frac{7}{8} - \frac{2}{8} = \frac{5}{8}$ **<u>Subtracting Fractions with denominators that are multiples of the same number</u>** Children are taught to use their knowledge of equivalent fractions to convert the fractions to the same denominator before subtracting them.  $\frac{5}{6}$ $-\frac{2}{3}$ $\frac{1}{6}$ For this they would first recognise that 2/3 is equivalent to 4/6 and then subtract 4/6 from 5/6. Children will use pictorial representations to support them with calculations. E.g. number lines, bar models and fraction walls.
Y5 ×	**<u>Multiply numbers mentally drawing upon known facts</u>**	Children will be taught to build upon their rapid recall of 1-12 x multiplication facts, and multiplication facts for multiples of 10 and 100 to calculate an increasing range of multiplication questions mentally. E.g. if they know 3x4 they can work out 30x4, 0.3x4 etc. Multiply a 2 or 3 digit number by a single digit by partitioning– e.g. 26 x 3 = 20 x 3 + 6 x 3 $6 \times 204 = 6\times200 + 6\times4$ $= 1{,}200 + 24$ $= 1{,}224$	

Multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit number

Multiply whole numbers and those involving decimals by 10, 100 and 1000

Multiply proper fractions and mixed numbers by whole numbers, supported by

Long Multiplication method

Children have been introduced to the formal written method of short multiplication for 2 or 3 digit numbers multiplied by one digit in year 4. This will be recapped prior to extending to long multiplication (see Yr 4 policy).

1	~~3~~	~~2~~	
	1	5	4
×		2	6
	9	2	4
3	0	8	0
4	0	0	4
1	1		

Start with the ones.

154 × 6 = 924

154 × 20 = 3080

3080 + 924 = 4004

N.B. Children are encouraged to use different colour pens for each line of working out if they struggle. See diagram for example of how colour can be used to show which digit the lines of working out relate to.

Multiplication by 10, 100 and 1000

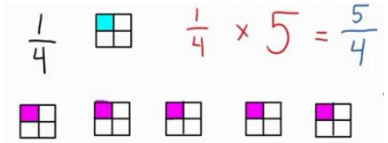
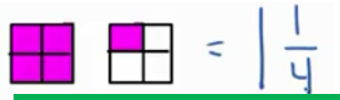
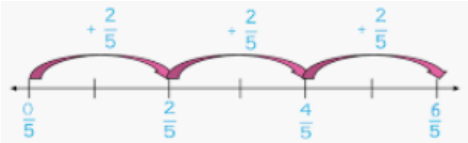
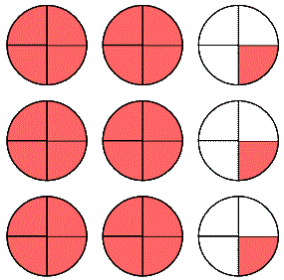
Children are provided with a laminated version of this grid to practise moving the digits when multiplying by 10, 100 and 1000. The majority of children will move on to drawing their own grid on their whiteboard in their book to support their calculations and then to complete the calculations mentally.

N.B. We continue to reiterate here that children **cannot** simply add a zero. When we work with numbers with decimal places, this becomes really apparent as the place value doesn't change, e.g. 5.6 is the same value as 5.60. The example in the table demonstrates the correct working for multiplying 5.6 by 10.

x10 — Move 1 place to the left for

x100 — Move 2 places to the left for

x1000 — Move 3 places to the left for

	materials and diagrams		**Multiply proper fractions and mixed numbers by whole numbers, supported by materials and diagrams**    Children are provided with visual representation to show how to multiply fractions. They are also taught how to convert an answer from an improper fraction to a mixed number as shown above. Number lines are used to show the repeated addition method for multiplying fractions. **Multiply mixed numbers by whole numbers**  Children are provided with visual representation to show how to multiply mixed numbers by whole numbers. They calculate using images to begin with. They are taught the following more abstract steps. 1. Convert the mixed number into an improper fraction. 2. Multiply the numerator by the whole number. 3. Convert the answer back into a mixed number by dividing the numerator by the denominator. The remainder is represented as a fraction. 2 ¼ x 3 = 4/4 + 4/4 + ¼ x 3 = 9/4 x 3 9/4 x 3 = 27/4 27 divided by 4 = 6 r3 6 3/4
Y5 ÷	**Divide numbers mentally drawing upon known facts**	Children will be taught to build upon their rapid recall of 1-12 x division facts, and dividing and multiplying by 10 and 100 to calculate an increasing range of division questions mentally. E.g. if they know 12 divided by 3 =4 they can work out 12 divided by 0.3= 40	

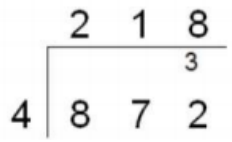
Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context

Divide whole numbers and those involving decimals by 10, 100 and 1000

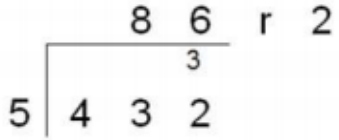
Divide numbers up to 4 digits by a one-digit number using the formal written method of short division

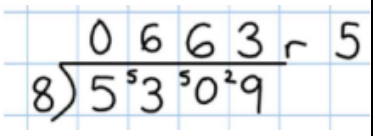
'Bus Stop Division' has been introduced in year 4 with 3 digit dividends and a single digit divisor with no remainders. This will be the first step in year 5. They will then move on to 3 digit dividends with single digit divisor with remainders. Finally, they will work with 4 digit dividends.

Begin with divisions that divide equally with no remainder.



Move onto divisions with a remainder.

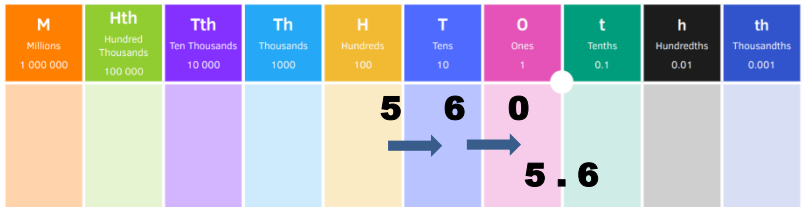




Interpreting remainders

Children will be taught how to interpret remainders from division questions and whether they should round to the next whole number or not. They will be taught to read questions carefully, underlining key words/phrases e.g. full boxes, how many do they need, how many … can be bought?

Division by 10, 100 and 1000



Children are provided with a laminated version of this grid to practise moving the digits when dividing by 10, 100 and 1000. The majority of children will move on to drawing their own grid on their whiteboard in their book to support their calculations and then to complete the calculations mentally.

N.B. We continue to reiterate here that children **cannot** simply remove zeros. Many of the numbers the children work with aren't multiples of 10 or 100 so they need to have the concept of the digits moving on the place value grid.

Move 1 place to the right for ÷10

Move 2 places to the right for ÷100

Move 3 places to the right for ÷1000

Year 6

UPPER KEY STAGE 2

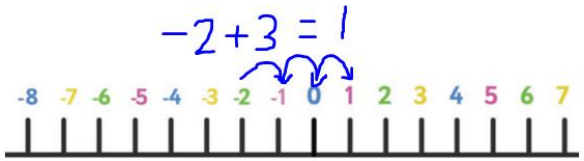
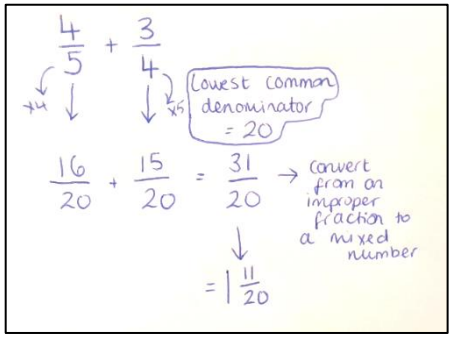
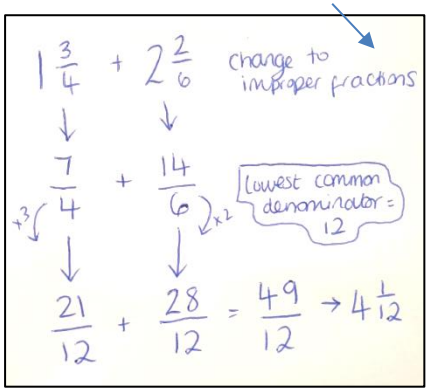
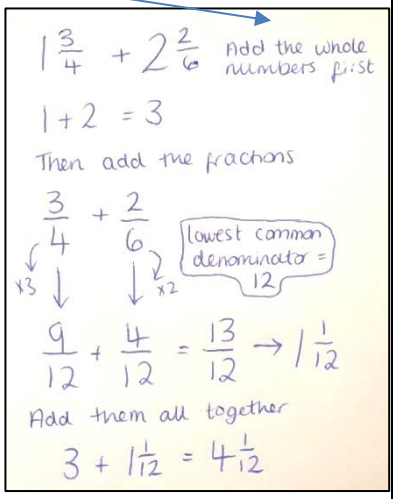
Children move on from dealing mainly with whole numbers to performing arithmetic operations with both decimals and fractions. By the end of year 6, pupils should be fluent in written methods for all four operations, including long multiplication and division, and in working with fractions, decimals and percentages.

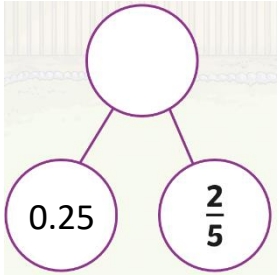
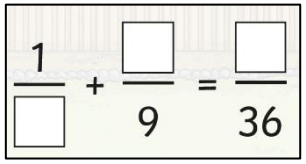
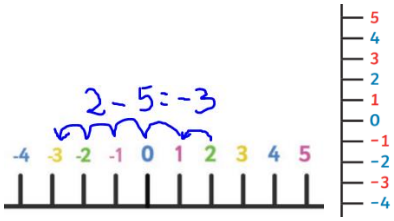
Addition and subtraction: Children will consolidate their use of written procedures in adding and subtracting whole numbers with up to 6 digits and also decimal numbers with up to 3 decimal places. Mental strategies for adding and subtracting increasingly large numbers will also be taught. These will draw upon children's robust understanding of place value and knowledge of number facts. Negative numbers will be added and subtracted.

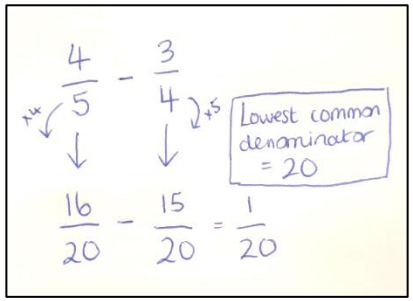
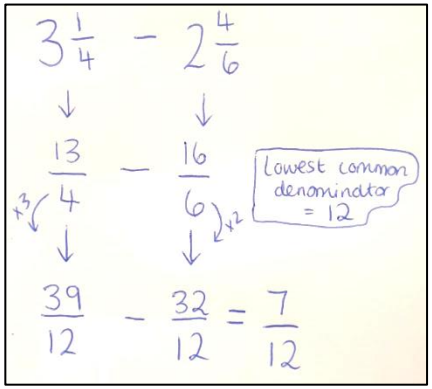
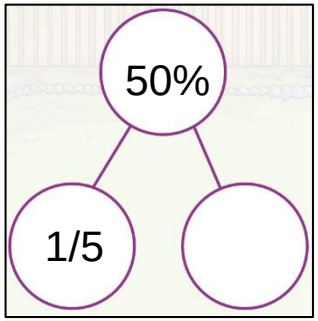
Multiplication and division: Efficient and flexible strategies for mental multiplication and division are taught and practised, so that children can perform appropriate calculations even when the numbers are large, such as 40 000 × 6 or 40 000 ÷ 8. In addition, it is in Years 5 and 6 that children extend their knowledge and confidence in using written algorithms for multiplication and division.

Fractions, decimals and percentages: Fractions and decimals are also added, subtracted, divided and multiplied, within the bounds of children's understanding of these more complicated numbers. Children will also calculate percentages and ratios.

	National Curriculum Objectives	Mental Calculation	Written Calculation- including concrete, pictorial and abstract methods
Y6 +	**<u>Undertake mental calculations with increasingly large numbers and more complex calculations</u>**	Have a focus on quick and accurate recall of number bonds to 100 (in ones and fives) and to 1000 (in hundreds and tens) and be able to apply these to larger numbers e.g. 51+49=100 so 510 +490 =1000 and therefore 5100 + 4900 = 10,000 etc. Encourage children to look for ways to simplify problems e.g. Money: £8.99 + £3.49 = £12.48 Use £9 + £3.50 = £12.50 and subtract 2p Children will be taught to count on from a	**<u>Column method for addition including regrouping</u>.** Children will be working with place value of numbers up to 10,000,000 in year 6 and will continue to build upon the column addition skills they have worked on in Y5 by calculating with numbers up to 6 digits 4 5 8 6 4 + 2 3 4 9 7 6 9 3 6 1 1 1 1 Starting with the ones, add each column in turn. Regroup tens, hundreds, thousands, ten thousands as required. 7 8 9 9 4 + 6 7 4 3 8 5 7 3 7 1 1 1 N.B. Children are encouraged to put their regrouped digit wherever they feel suits them best. They are shown different ways and are allowed to choose Children will also use this method to add numbers that have up to 3 decimal places N.B. Children are given problems which involve adding numbers with differing place value and involving whole numbers added to numbers with decimal places. We teach children to use place holders to help them to line their digits up with the correct place value.

	Use negative numbers in context and calculate intervals across zero.	negative number up through zero in ones and to do this with problems in context.	**Calculating negative numbers pictorially-** Children are encouraged to draw number lines to help them to calculate intervals through zero. They are shown number lines both horizontally and vertically, also in context using thermometers. They can then use these number lines to make 'jumps' as they have done in previous year groups so help them to see the changes as they cross zero.  
	Add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions	Use common factors to simplify fractions mentally	**Adding fractions** Children are taught to change the fractions to an alternate equivalent fraction so that they both have the same denominator, add the numerators and then simplify or change to a mixed number if needed e.g. When adding mixed numbers, we teach the children these two methods.   

			<u>Use of the part-whole model for adding fractions, decimals and percentages</u>  Children have use part-whole models all through school. We use them in many different contexts in year 6, here is one example- we use them to get the children to practise converting decimals, fractions and percentages to the same thing and then adding them. They choose the best way to convert before adding.  Missing number problems are used to help support reasoning and problem solving
Y6 -	**<u>Use negative numbers in context and calculate intervals across zero.</u>**	Children will be taught to <u>count back</u> through zero in ones and to do this with problems in context.	**<u>Calculating negative numbers pictorially</u>-** Children are encouraged to draw number lines to help them to calculate intervals through zero. They are shown number lines both horizontally and vertically, also in context using thermometers. They can then use these number lines to make 'jumps' as they have done in previous year groups so help them to see the changes as they cross zero.  **<u>Column method for subtraction including exchanging.</u>** Children will be working with numbers up to 10,000,000 in year 6 and will continue to build upon the column subtraction skills they have worked on in Y5 by calculating with numbers containing up to 6 digits 3 5 ~~7~~ ~~4~~ ~~2~~ - 3 4 7 6 3 2 2 6 6 Starting with the ones, subtract each column in turn. Exchange tens, hundreds, thousands and/or ten thousands as required. N.B. Children are also exposed to tricky calculations where the larger number is a multiple of 10,000 so they have to use and apply their knowledge of exchanging to solve it.

	Subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions	Use common factors to simplify fractions mentally	**Subtracting Fractions** Children are taught to change the fractions to an alternate equivalent fraction so that they both have the same denominator, subtract the numerators and then simplify or change to a mixed number if needed e.g.  When subtracting with mixed numbers, we teach the children to convert the mixed numbers to improper fractions first and then subtract as they can't always subtract the whole numbers first.  **Use of the part-whole model for subtracting fractions, decimals and percentages**  Children have use part-whole models all through school. We use them in many different contexts in year 6, here is one example- we use them to get the children to practise converting decimals, fractions and percentages to the same thing and then subtracting them. They choose the best way to convert before subtracting.
Y6 ×	**Perform mental calculations, including with mixed operations and large numbers**	Encourage children to think about the order in which they calculate, e.g. **Order of calculations:** 50 × 34 × 2 = 50 × 2 × 34 = 100 × 34 = 3400	

Multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication

Multiply simple pairs of proper fractions, writing the answer in its simplest form

N.B. Children are taught that **of** and **x** are interchangeable in these types of calculations e.g. 2/5 **x** 3 is the same as 2/5 **of** 3

Long Multiplication method

1	~~3~~	~~2~~	
	1	5	4
×		2	6
	9	2	4
3	0	8	0
4	0	0	4
1	1		

Start with the ones.

154 × 6 = 924

154 × 20 = 3080

3080 + 924 = 4004

N.B. This method has been introduced in year 5 so they should be familiar with it. We focus on SATs style arithmetic questions and making sure children check their working by repeating the calculation to check they get the same answer or doing the inverse.

N.B. Children are encouraged to use different colour pens for each line of working out if they struggle. See diagram for example of how colour can be used to show which digit the lines of working out relate to.

124 × 26 becomes

	1	2	
	1	**2**	**4**
×		**2**	**6**
	7	**4**	**4**
2	**4**	**8**	**0**
3	**2**	**2**	**4**
1	1		

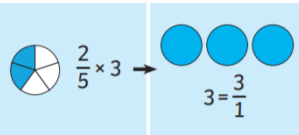
Answer: 3224

Multiplying Fractions

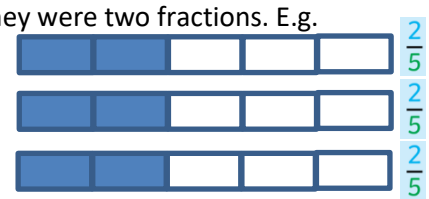
$$\frac{1}{2}\times\frac{1}{3}=\frac{1}{2}\times\frac{1}{3}=\frac{1}{6}$$

Multiply the numerators together, multiply the denominators together, simplify or change to a mixed number if needed

Children will also multiply proper fractions by whole numbers. We teach the children to change the whole number to become a fraction over 1 and multiply as if they were two fractions. E.g.



$$\frac{2}{5}\times\frac{3}{1}=\frac{6}{5}=1\frac{1}{5}$$



$$=\frac{6}{5}=1\frac{1}{5}$$

We use bar models and diagrams like the ones above to support the teaching of this. The bar model and diagrams support the repeated addition of the fractional parts.

Identify the value of each digit in numbers given to three decimal places and multiply numbers by 10, 100 and 1000 giving answers up to three decimal places

Multiply one-digit numbers with up to two decimal places by whole numbers

Use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy.

Children will often use estimation to check the reliability of their answers for multiplication and division. We encourage children to estimate the answers first by rounding, so 3.19 x 12, they would round the decimal number to the nearest whole, 3x12=36. They also need to check that their decimal point in their answer box lines up with the one in the question.

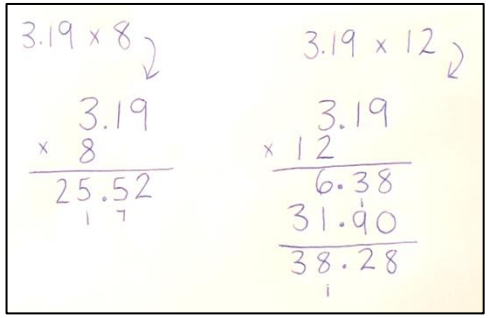
Multiplication by 10, 100 and 1000

Children are provided with a laminated version of this grid to practise moving the digits when multiplying by 10, 100 and 1000. The majority of children will move on to drawing their own grid on their whiteboard in their book to support their calculations and then to complete the calculations mentally.

Move 1 place to the left for x10
Move 2 places to the left for x100
Move 3 places to the left for x1000

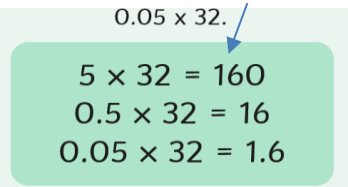
N.B. We continue to reiterate here that children **cannot** simply add a zero. When we work with numbers with decimal places, this becomes really apparent as the place value doesn't change, e.g. 5.6 is the same value as 5.60. The example in the table demonstrates the correct working

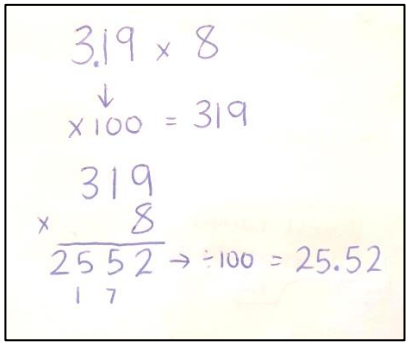
Short and long multiplication of one-digit numbers with up to two decimal places and whole numbers



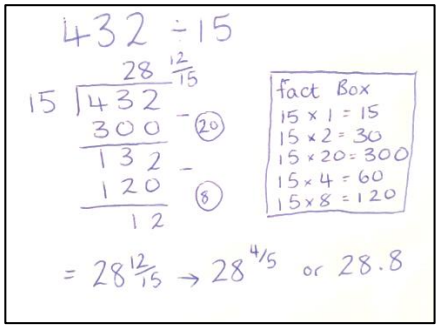
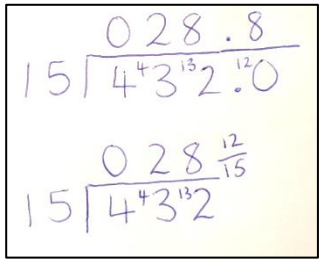
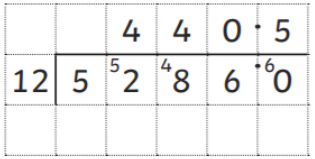
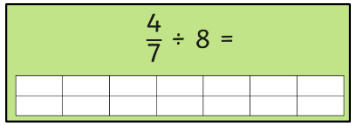
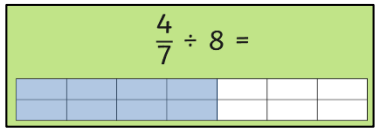
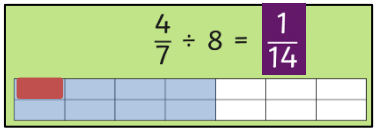
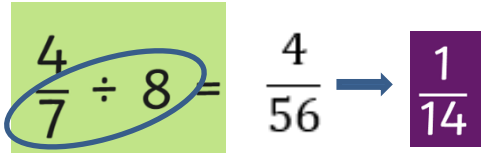
Children will use the same method of short or long multiplication as they would with whole numbers and will also use place value to make sure the digits are lined up correctly.

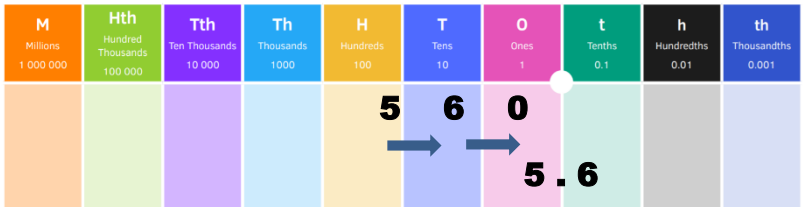
Children can use multiplication facts to help them e.g.



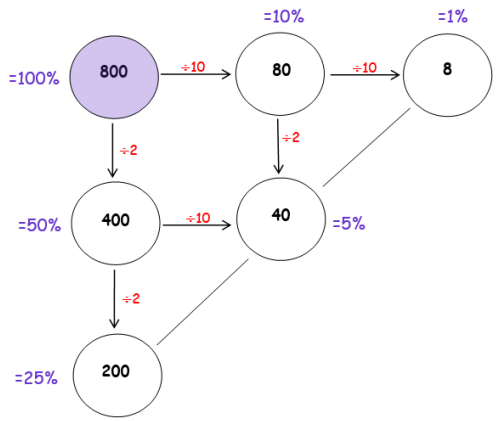


Children can also multiply the number out to get a whole number and work the calculation through, then divide the answer by the same amount.

Y6 **÷**	**Divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context** **Use written division methods in cases where the answer has up to two decimal places** **Divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context**	**Perform mental calculations, including with mixed operations and large numbers** Children are encouraged to use their knowledge of division facts to help them with calculating with larger numbers e.g. For 5400 ÷ 6, they can use 54÷6=9 540÷6=90 So 5400÷6=900	**Long Division- Chunking** In year 6, children are taught to show remainders of division calculations as **fractions** or **decimals**.  Children create a fact box for the divisor. They don't need to include every multiple of that number, only ones that are relevant to the calculation. It is sometimes easier to create the fact box as they are going along. These chunks are then subtracted from the dividend until they can no longer remove a whole chunk or get to zero. Any amount left over is the remainder. This remainder then needs to be interpreted as a fraction or decimal. **Short Division**  Children may still choose to create a fact box depending on the size of the dividend and divisor. They use the short method of division starting from the highest value digit in the divisor. If the child is interpreting the remainder as a decimal, they will need to use a place holder after the decimal point and continue to divide. They can also interpret their remainder as a fraction.  **Divide proper fractions by whole numbers** We begin by using bar models and diagrams to show how the fraction is divided    Once the children understand how the fractional part is divided, we use an abstract method to allow them to reach the answer more quickly and efficiently. 1. Keep the numerator the same 2. Multiply the denominator by the whole number to become the new denominator 3. Simplify if needed 

	Identify the value of each digit in numbers given to three decimal places and divide numbers by 10, 100 and 1000 giving answers up to three decimal place		**Division by 10, 100 and 1000**  Children are provided with a laminated version of this grid to practise moving the digits when dividing by 10, 100 and 1000. The majority of children will move on to drawing their own grid on their whiteboard in their book to support their calculations and then to complete the calculations mentally. Move 1 place to the right for ÷10 Move 2 places to the right for ÷100 Move 3 places to the right for ÷1000 N.B. We continue to reiterate here that children **cannot** simply remove zeros. Many of the numbers the children work with aren't multiples of 10 or 100 so they need to have the concept of the digits moving on the place value grid
	Associate a fraction with division and calculate decimal fraction equivalents		**Relating division to fractions** Show children that the division symbol is actually very similar to a fraction but without numbers as numerator and denominators. Children need to understand that fractions are related to division e.g. ½ is the same as 1÷2 $\frac{3}{8} = 8\overline{)3.^{3}0^{6}0^{4}0}$ (0.375) $\frac{1}{2} = 2\overline{)1.^{1}0}$ (0.5) $\frac{1}{5} = 5\overline{)1.^{1}0}$ (0.2)
	Use their knowledge of the order of operations to carry out calculations involving the four operations (BODMAS)		B – Brackets: $10 \times (4 + 2) = 10 \times 6 = 60$ O – Order: $5 + 2^2 = 5 + 4 = 9$ D – Division: $10 + 6 \div 2 = 10 + 3 = 13$ M – Multiplication: $10 - 4 \times 2 = 10 - 8 = 2$ A – Addition: $10 \times 4 + 7 = 40 + 7 = 47$ S – Subtraction: $10 \div 2 - 3 = 5 - 3 = 2$ N.B. The O in BODMAS is also referred to as 'of' as in 'powers of' and an I for indices. Pupils explore the order of operations using brackets; for example, 2 + 1 x 3 = 5 and (2 + 1) x 3 = 9.

%	**Solve problems involving the calculation of percentages [for example, of measures, and such as 15% of 360] and the use of percentages for comparison**		**The Bubble Method**



To find a percentage of any number:

Children fill in the value of each circle, beginning with the main number in the shaded area. They then work their way through all 6 circles by following the actions on each arrow. They can then use the information in each circle to find any percentage.

e.g. 76% of 800, you would add

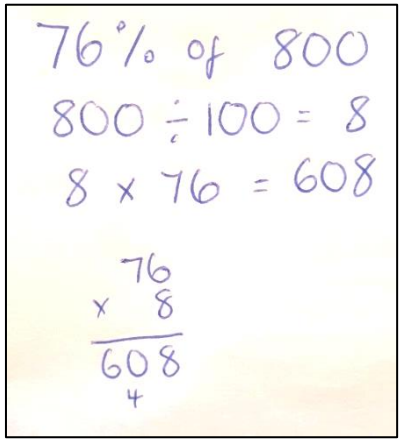
50%= 400

25%=200

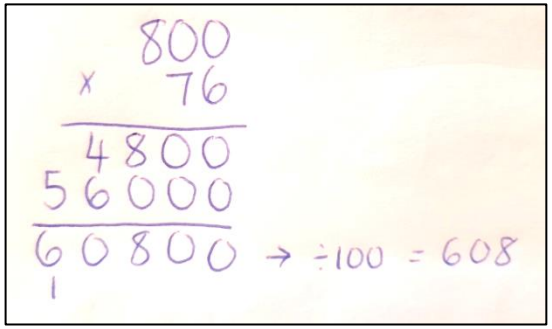
1%=8

76% = 608

Divide by 100 and then multiply by the percentage

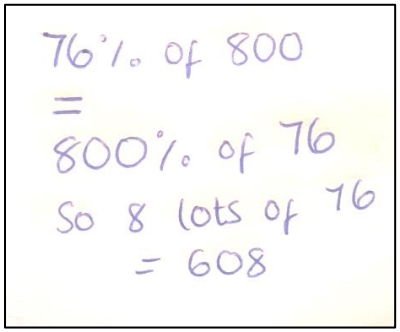


Multiply by the percentage and divide by 100

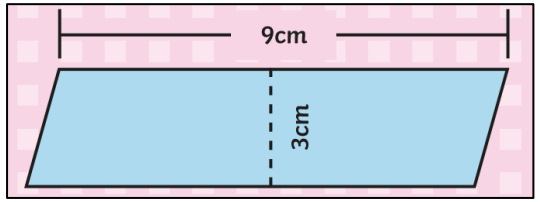
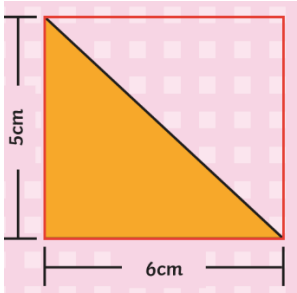


Flip the values

(if it easier to do so- some values it wouldn't make sense to do this)



N.B. Children are taught all 4 methods and then they choose the method that they are most comfortable with to solve calculations

Shape and Measure	**Convert between miles and kilometres** **Calculate the area of parallelograms and triangles**		5 miles ≈ 8 kilometres Children are taught that 1 mile is approximately 1.6km. The whole number equivalent is 5 miles approximately equals 8km. Miles to Kilometres- Multiply by 8 then divide by 5 Kilometres to miles- Multiply by 5 then divide by 8 Alternatively, children can multiply or divide by 1.6 if they are confident. **Area of parallelograms and triangles** Area of a parallelogram = length × perpendicular height  9cm x 3cm = 27cm^2 Area of a triangle = (base × height) ÷ 2  6cm x 5cm = 30cm ÷ 2 = 15cm^2